

IMPROVING LINEAR PROGRAMMING MODEL FOR ASSESSING PRODUCT SALES TAKING INTO ACCOUNT SOCIAL RESPONSIBILITY OF CONSUMERS

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The purpose of this study is to improve the linear programming model, designed for evaluating sales of product variants with different social utility to consumers with different social responsibility, by taking into account the real conditions for making purchases. This will make it possible to obtain more realistic estimates of future sales of socially beneficial products.

To achieve the goal, the following tasks were set:

- in the considered linear programming model, formalize the set of consumer groups with different preferences for the set of analyzed goods;
- formalize in the model certain aspects of shopping by consumers with different social responsibility (in particular, devise a way to take into account in the model a realistic sequence of purchases over time, as well as the behavior of buyers who equally evaluate the usefulness of several product options);
- test the improved model on the example of the office paper market.

The distribution of volumes of product options between buyers, which is optimal according to the criterion of the maximum total utility of purchases in the market, can be determined using a linear programming model:

$$\left\{ \begin{array}{l} \sum_i \sum_j u_i^j \cdot x_i^j \rightarrow \max, \\ \sum_j x_i^j \leq v_i, i = \overline{1, n}, \\ \sum_i x_i^j \leq d_j, j = \overline{1, m}, \\ x_i^j \geq 0, \end{array} \right. \quad (1)$$

x_i^j – the volume of sales of the i -th variety of products to buyers of the j -th group;

u_i^j – ordinalistic evaluation of the utility of the i -th variety of goods from the point of view of buyers of the j -th group;

v_i – the volume of supply of the i -th variety of products;

d_j – the volume of demand for the analyzed type of products from buyers of the j -th group

1. Formalization of the set of consumer groups with different preferences for the set of analyzed goods

This research is based on the assumption that the choice made by the consumer on a set of goods can be uniquely described by the vector of product ranks. The rank vector actually specifies the procedure for choosing a product by the consumer: among the set of alternative products, the consumer chooses the product with rank 1; in the absence of a product with rank 1, chooses a product with rank 2, etc. The vector of ranks of goods for each consumer can be determined by means of a questionnaire of the type of ranking. Consumers who have the same rank vector on a given set of goods are combined into one group.

Under the adopted assumptions, the total set of consumer groups can be determined by sorting through all possible variants of the product rank vector. To reduce the number of consumer groups under consideration, it is recommended to limit to those groups in which the rank vectors correspond to the rules of consumer logic.

Table – Variants of the rank vector and corresponding groups of consumers for the case of four types of goods, excluding rank vectors with repeated ranks

Product rank	Group of consumers																							
	g ₁	g ₂	g ₃	g ₄	g ₅	g ₆	g ₇	g ₈	g ₉	g ₁₀	g ₁₁	g ₁₂	g ₁₃	g ₁₄	g ₁₅	g ₁₆	g ₁₇	g ₁₈	g ₁₉	g ₂₀	g ₂₁	g ₂₂	g ₂₃	g ₂₄
r _{1j}	1	2	3	4	1	2	3	4	1	1	1	1	2	4	3	4	4	2	3	4	3	3	2	2
r _{2j}	2	1	4	3	3	4	1	2	2	4	3	4	1	1	1	1	3	3	2	2	4	2	4	3
r _{3j}	3	4	1	2	2	1	4	3	4	2	4	3	3	3	2	2	1	1	1	1	2	4	3	4
r _{4j}	4	3	2	1	4	3	2	1	3	3	2	2	4	2	4	3	2	4	4	3	1	1	1	1

2. Formalization in the model of the behavior of buyers who equally evaluate the usefulness of several product options

This advancement is based on the assumption that buyers who equally evaluate the usefulness of several product options choose one of the equivalent product options randomly, with the same probability for each option.

Since in the linear programming problem (1) consumer choice is not determined by means of probability theory but by means of utility theory, we propose to describe random consumer choice by the same means.

*To formalize random selection, we suggest using **fictitious groups** of buyers.*

For example, if buyers from group k with a quantity of d equally evaluate g product variants $a_1, a_2, \dots, a_g$, then it can be assumed that as a result of random selection, buyers in the quantity of d/g will buy product variant a_1 , the same quantity of other buyers will buy product a_2 and etc. More precisely, the quantity of buyers of each product will approach the value of d/g as the quantity of the group increases. Therefore, to simulate a random choice, it is advisable to divide the consumer group k into g fictitious groups, in each of which one of the product options is preferred. But this is not enough. In the considered model, for each consumer group, not only the preferred product is specified but a full vector of ranks, which specifies the procedure for choosing a product by the consumer in the absence of some products. If there is no product with rank 1, consumers will buy the product with rank 2; if this is not there, then they will buy a product with rank 3, etc. Therefore, an additional assumption was introduced: each buyer from the consumer group k , which corresponds to a vector of ranks with the same evaluations of products $a_1, a_2, \dots, a_g$, chooses one in the process of purchase of g items at random. At the same time, s/he demonstrates behavior that is described by one of the possible options for strict ranking of these goods. There is a total of $g!$ such options. Based on this, to simulate random selection, it is advisable to divide the members of the group k into $g!$ subgroups, the number of each of which approaches $d/g!$ where d is the size of group k .

3. Taking into account the sequence of purchases over time in the model

In order to take into account in the model a realistic sequence of sales over time, the assumption is accepted that purchases are made with a constant intensity during the entire time period T , which is considered, and consumers from different groups make purchases in parallel in time. It is also assumed that consumers from different groups receive goods from the same sources, and therefore, if a product runs out, it will run out simultaneously for all groups of buyers.

If the total volume V of the supply of the product is less than the total volume D of demand for it and is equal to $\alpha \cdot D$, then the supply of this product will be exhausted in the period $\alpha \cdot T$.

Then the demand from each group of buyers will be satisfied by this product at level α :

$$\frac{\sum_{i=1}^n x_i^j}{d_j} = \alpha, \quad j = \overline{1, m}, \quad \text{where } \alpha = \begin{cases} \frac{V}{D} = \frac{\sum_{i=1}^n v_i}{\sum_{j=1}^m d_j}, & \text{if } V < D, \\ 1, & \text{if } V \geq D. \end{cases} \quad (2)$$

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The improved was tested on the example of modeling the structure of sales in the office paper market taking into account the environmental responsibility of consumers.

In the office paper market, there are two interchangeable versions of the product – "ordinary paper" and "environmentally-friendly paper". These product options differ in price and environmental friendliness. According to the price criterion, the usual version of the product is better for buyers. According to the criterion of sustainable development, the environmentally-friendly option is the best. According to the quality criterion, there are no significant differences between the options if they belong to the same class of paper (class A, B, or C). An analysis of the B-class office paper offer on the Rozetka marketplace revealed that 60 % of the office paper brands have an FSC or PEFC certificate, as well as the Ecolabel marking, that is, they can be categorized as eco-goods.

To test the model, a case was considered when the new company A plans to enter the equilibrium market of office paper, in which the volume of supply equals the volume of demand. Company A plans to offer eco-paper and ordinary paper in the market, each product in the amount of 10 % of the total supply of goods by other companies.

Thus, the analyzed market includes the following product variants: product 1 – eco-paper from company A; product 2 – eco-paper from other enterprises; product 3 – ordinary paper from enterprise A; product 4 – ordinary paper from other enterprises. If all volumes of goods are evaluated as a percentage of the volume of demand, then, based on the initial data and taking into account the results of the "Rozetka" analysis, I have the following volumes of supply of goods in the market: $v_1=10.0$; $v_2=60.0$; $v_3=10.0$; $v_4=40.0$.

The task was to build a model that would make it possible to estimate the volume of products that consumers would buy from the analyzed company A, considering the given market characteristics and buyer preferences close to actual ones.

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Fig. A decision tree that describes the logic of consumer choice for multiple options of office paper, as well as corresponding groups of consumers with the same preferences

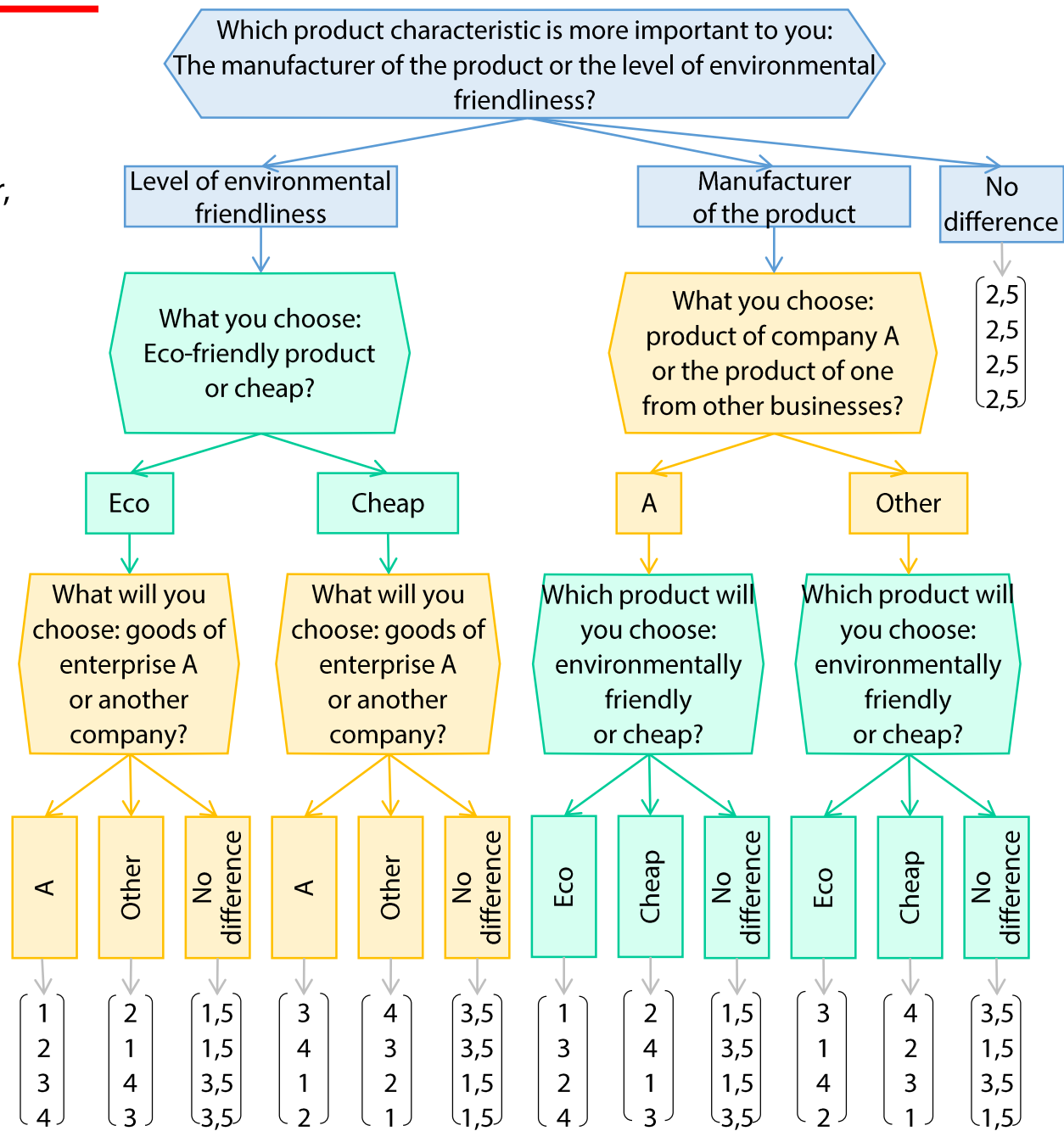


Table – Structure of the division of respondents into consumer groups revealed by questionnaires

Group	Rank vector	Description	Percentage of respondents
b_1	(2 1 4 3)	Consumers of this group choose paper, first of all, on the grounds of environmental friendliness; in addition, they show loyalty to one of the paper manufacturers	6.5
b_2	(4 3 2 1)	Consumers of this group choose paper, first of all, on the basis of cheapness; in addition, they show loyalty to one of the paper manufacturers	0
b_3	(3 1 4 2)	Consumers of this group choose, first of all, paper of a specific manufacturer; among paper options from your favorite manufacturer, choose an ecological option	8.8
b_4	(4 2 3 1)	Consumers of this group choose, first of all, paper of a specific manufacturer; choose a cheap option among paper options from your favorite manufacturer	4.3
b_5	(1.5 1.5 3.5 3.5)	Consumers of this group choose paper based on environmental friendliness; the manufacturer is not important for them	43.5
b_6	(3.5 3.5 1.5 1.5)	Consumers of this group choose cheap paper; the manufacturer is not important for them	13.0
b_7	(3.5 1.5 3.5 1.5)	Consumers of this group choose paper of a specific manufacturer; "environmental-price" characteristics are not important for them	2.2
b_8	(2.5 2.5 2.5 2.5)	Consumers of this group choose paper randomly, without paying attention to price and environmental characteristics	21.7

Based on the given input data, the initial (unmodified) linear programming model designed to evaluate the structure of sales of the analyzed goods with different social utility takes the following form:

$$\left\{ \begin{array}{l} \sum_i \sum_j u_i^j \cdot x_i^j \rightarrow \max, \\ \sum_j x_1^j \leq 10.0, \quad \sum_j x_2^j \leq 60.0, \quad \sum_j x_3^j \leq 10.0, \quad \sum_j x_4^j \leq 40.0, \\ \sum_i x_i^1 \leq 6.5, \quad \sum_i x_i^2 \leq 0, \quad \sum_i x_i^3 \leq 8.8, \quad \sum_i x_i^4 \leq 4.3, \\ \sum_i x_i^5 \leq 43.5, \quad \sum_i x_i^6 \leq 13.0, \quad \sum_i x_i^7 \leq 2.2, \quad \sum_i x_i^8 \leq 21.7, \\ x_i^j \geq 0, \\ i = \overline{1,4}, \quad j = \overline{1,8}. \end{array} \right. \quad (3)$$

The modified model of linear programming taking into account fictitious groups of consumers takes the following form:

$$\left\{ \begin{array}{l} \sum_i \sum_j u_i^j \cdot x_i^j \rightarrow \max, \\ \sum_j x_1^j \leq 10.0, \quad \sum_j x_2^j \leq 60.0, \quad \sum_j x_3^j \leq 10.0, \quad \sum_j x_4^j \leq 40.0, \\ \sum_i x_i^j = \alpha \cdot d_j, j = \overline{1, 24}, \quad \text{de} \quad \alpha = \frac{\sum_{i=1}^n v_i}{\sum_{j=1}^m d_j} = 1, \\ x_i^j = \beta_i \cdot d_j, j \in J_i, \quad \text{de} \quad \beta_i = \frac{v_i}{\sum_{j \in J_i} d_j}, \quad i = \overline{1, 4}, \\ x_i^j \geq 0. \end{array} \right. \quad (4)$$

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Problem (4) was solved by the simplex method implemented by Python tools.
After the reverse transition from fictitious consumer groups to real ones, a solution to the initial problem with 4 producers and 13 consumer groups was obtained. The vector of total sales of four types of paper is equal to (10.0 59.0 10.0 20.5).

Table – Solutions to the given problem of linear programming with fictitious groups

Sales volume	g ₁	g ₂	g ₃	g ₄	g ₅	g ₆	g ₇	g ₈	g ₉	g ₁₀	g ₁₁	g ₁₂	g ₁₃	g ₁₄	g ₁₅	g ₁₆	g ₁₇	g ₁₈	g ₁₉	g ₂₀	g ₂₁	g ₂₂	g ₂₃	g ₂₄
x _{1j}	4.3	0.0	0.0	0.0	0.3	0.0	0.0	0.0	4.3	0.3	0.3	0.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
x _{2j}	7.4	18.3	0.0	0.0	0.6	0.0	10.3	0.0	7.4	0.0	0.0	0.0	11.8	1.5	0.9	0.9	0.0	0.1	0.1	0.1	0.0	0.0	0.0	0.0
x _{3j}	0.0	0.0	3.5	0.0	0.0	0.8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	3.5	0.8	0.8	0.8	0.0	0.0	0.0	0.0
x _{4j}	0.0	0.0	0.7	4.2	0.0	0.1	0.0	5.8	0.0	0.6	0.6	0.6	0.0	0.0	0.0	0.0	0.7	0.0	0.0	0.0	4.2	1.5	0.9	0.9

Fig. Results of calculations: distribution of volumes of goods among buyers, which is optimal according to the criterion of maximum total utility for the totality of buyers in the market



1. A set of consumer groups with different preferences has been formalized in the linear programming model designed to estimate sales volumes in the market taking into account the aspect of social responsibility. It is proposed to distinguish consumer groups on the basis of product rank vectors, which characterize the preferences of each consumer on the set of analyzed products and are revealed by questionnaires. Consumers who have the same product rank vector are united into one group. The coefficients of the total sales utility function are also calculated based on the values of the rank vectors. The total set of consumer groups in a specific market is recommended to be formed on the basis of a consumer decision tree.
2. The formalization of certain aspects of shopping by consumers with different social responsibilities has been carried out in the considered model. First, the behavior of buyers who treat several goods with different social utility and choose a product randomly is formalized. To take into account in the model the random choice of such consumers, their groups are divided into subgroups, each of which corresponds to one of the possible options for the consumer's behavior regarding the choice of products.
Secondly, the model takes into account a realistic sequence of purchases in time, according to which a specific unit of the product is bought by the buyer who previously chose it, and not by the one who values it higher. This is done with the help of additional constraints in the form of equations that describe the fact that each product will run out simultaneously for all groups of consumers.
3. The improved model was tested on the example of simulation of sales in the office paper market, which includes two general types of goods – environmentally-friendly paper and paper that does not meet modern environmental requirements.