Integral Inequalities in Macroeconomic Dynamics

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The construction of mathematical models significantly improves our understanding of the nature of economic processes and increases the accuracy of forecasting results. Particular attention is paid to dynamic models, which enable us to study transient phenomena, assess the stability limits of a system's equilibrium state, and predict the occurrence of bifurcations and socioeconomic cataclysms.

The aim of the study is to construct a mathematical model of the dynamics of changes in national income over time, taking into account the memory effect of the system.

Classical Model of Macroeconomic Systems

The classical equation that characterizes the balance between supply and aggregate demand is:

$$Y = C + I + G,$$

where Y is national income volume;

C is consumption volume;

I is investment;

G is independent government spending.

Proposed Hypothesis

We proposed a hypothesis that the peculiarity of the dynamics of macroeconomic processes is such that for any moment in time, supply $Y(t)$ does not exceed aggregate demand $C(t) + I(t) + G(t)$.

Instead of equality, we consider inequality:

$$Y(t) \leq C(t) + I(t) + G(t),$$

where t is a point of time for which the state of the system is considered.

Model Assumptions

1. Investment $I(t)$ does not depend on national income $Y(t)$.

2. We have exogenous variable $A(t) = I(t) + G(t)$.

3. There is a continuously distributed lag on the consumption side from income and this lag depends on the previous values of the consumption function $C(t)$:

$$C(t) = c\int_0^t K(t,\tau)Y(\tau)d\tau,$$

where c is the marginal propensity to consume;

$K(t,\tau)$ Is a dynamic memory factor.

Basic Macroeconomic Inequality in the Form of Integral Inequality

$$Y(t) \leq c\int_{0}^{t} K(t,\tau)Y(\tau)d\tau + A(t)$$

where τ is a moment in time from the period that precedes the moment in question.

$K(t,\tau)$ has the meaning of a dynamic memory of the past values of the process under study. It is a decreasing function of time.

τ in relation to the moment

Special Case: Degeneracy of the Core

In the core is $K(t,\tau)=\alpha(t)\cdot\beta(\tau)$ fundamental inequality of the model takes the form:

$$Y(t)\leq c\cdot\alpha(t)\int_{0}^{t}\beta(\tau)Y(\tau)d\tau+A(t)$$

This inequality is essentially a special case of the inequality known as Gronwall–Bellman Inequality:

$$Y(t)\leq c\cdot\alpha(t)\int_{0}^{t}A(\tau)\beta(\tau)\exp\left(c\int_{\tau}^{t}\alpha(\xi)\beta(\xi)d\xi\right)d\tau+A(t)$$

Where is a moment in time from the period that belongs to a subsequent moment in time in relation to the moment but preceding the moment in time under consideration.

Dynamic Memory Factor: Exponential Form

Let's assume that $\alpha(t) = e^{-\lambda t}$ and $\beta(t) = \lambda e^{\lambda t}$

where λ is a parameter that characterizes the rate of "forgetting" of the state of the system at past moments and this decrease is described by an exponential dependence. Accordingly, we obtain:

$$Y(t) \le c \cdot \lambda \int_0^t A(\tau) \cdot e^{-(1-c)\lambda(t-\tau)} d\tau + A(t)$$

Economic Growth Result

If $1-c=s$, we obtain:

$$Y(t)\le c\cdot\lambda\int_0^t A(\tau)\cdot e^{-s\lambda(t-\tau)}d\tau + A(t)$$

where s parameter, which describes the marginal propensity to accumulate.

If $A(t)=A_0=const$, this simple inequality takes place:

$$Y(t)\le\frac{A_0}{s}\left(1-c\cdot e^{-s\lambda}\right)$$

Economic Growth Result

Let's analyze the inequality:

$$Y(t) \le \frac{A_0}{s}\left(1 - c \cdot e^{-s\lambda}\right).$$

It demonstrates how the income value evolves from the initial value $Y(t = 0) = A_0$ to the equilibrium value $Y(t \to \infty) = s^{-1} \cdot A_0$. This is a result of the multiplier effect in the Keynesian model.

Since $s < 1$, then in this case economic growth takes place.

Practical Significance

Thus, the use of a mathematical model of macroeconomic dynamics to describe changes in income over time allows us to estimate the limit of the state's economic potential. The model allows:

- better forecasting of national income;
- useful for policy-making and stabilization programs;
- helps predict crises and socio-economic transitions.

Thank You

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